

Field-Oriented Control of PMSMs

Clarke and Park written out, the dq machine model, current-loop tuning, MTPA, and field weakening.

1. Clarke: three phases to a stationary two-axis frame

Amplitude-invariant form (a balanced phase current of peak I becomes a vector of length I):

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}.$$

With $i_a + i_b + i_c = 0$ only two phases need measuring: $i_\alpha = i_a$, $i_\beta = (i_a + 2i_b)/\sqrt{3}$. (The power-invariant variant uses $\sqrt{2/3}$ instead of $2/3$, which changes the $\frac{3}{2}$ in the torque equation below. Check which convention a library uses.)

2. Park: rotate into the rotor frame

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos \theta_e & \sin \theta_e \\ -\sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}, \quad \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} \cos \theta_e & -\sin \theta_e \\ \sin \theta_e & \cos \theta_e \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix}.$$

In complex notation, $i_d + ji_q = e^{-j\theta_e}(i_\alpha + ji_\beta)$. A balanced set gives $i_\alpha + ji_\beta = Ie^{j(\theta_e + \varphi)}$, so $i_d + ji_q = Ie^{j\varphi}$: a **constant**. That is the whole trick. Here $\theta_e = p\theta_m$ with p pole pairs.

3. The PMSM in dq coordinates

$$\begin{aligned} v_d &= R i_d + L_d \frac{di_d}{dt} - \omega_e L_q i_q, \\ v_q &= R i_q + L_q \frac{di_q}{dt} + \omega_e L_d i_d + \omega_e \psi_m, \\ T_e &= \frac{3}{2} p [\psi_m i_q + (L_d - L_q) i_d i_q]. \end{aligned}$$

The $\omega_e L$ terms are **speed-dependent cross-coupling**: the d-axis current disturbs the q-axis voltage and vice versa, and $\omega_e \psi_m$ is the back-EMF. Surface-magnet (SPM) machines have $L_d \approx L_q$, so torque is just $\frac{3}{2} p \psi_m i_q$. Interior-magnet (IPM) machines have $L_q > L_d$, which adds reluctance torque when $i_d < 0$.

4. The current loop

Decoupling feedforward lets each PI see a simple RL plant:

$$v_d^* = \text{PI}_d(i_d^* - i_d) - \omega_e L_q i_q, \quad v_q^* = \text{PI}_q(i_q^* - i_q) + \omega_e (L_d i_d + \psi_m).$$

With plant $1/(R + sL)$, choose $K_p = \omega_c L$ and $K_i = \omega_c R$. The PI zero cancels the plant pole and the closed loop becomes $\omega_c/(s + \omega_c)$, first order with bandwidth ω_c . Keep ω_c well below the sampling rate (often a tenth or less of $2\pi f_s$), because PWM and computation add about 1.5 samples of delay.

Inverse Park produces v_α^*, v_β^* , and space-vector modulation turns them into gate signals. In the linear SVM range the largest achievable vector is $V_{\max} = V_{\text{dc}}/\sqrt{3}$, in the amplitude-invariant convention.

5. Limits, MTPA, and field weakening

Two constraints bound every operating point:

$$i_d^2 + i_q^2 \leq I_{\max}^2 \quad (\text{current circle}), \quad (L_q i_q)^2 + (L_d i_d + \psi_m)^2 \leq \left(\frac{V_{\max}}{\omega_e} \right)^2 \quad (\text{voltage ellipse})$$

The voltage ellipse is centered at $i_d = -\psi_m/L_d$ and **shrinks as speed rises**. Below base speed it contains the optimal point. Above base speed you must move along it toward negative i_d : that is **field weakening**.

Maximum torque per amp (IPM)

With $\Delta L = L_q - L_d > 0$, minimizing $i_d^2 + i_q^2$ at fixed torque (Lagrange multiplier) gives

$$\Delta L i_d^2 - \psi_m i_d - \Delta L i_q^2 = 0 \Rightarrow i_d = \frac{\psi_m - \sqrt{\psi_m^2 + 8 \Delta L^2 I_s^2}}{4 \Delta L}, \quad I_s = \sqrt{i_d^2 + i_q^2}.$$

The root is always negative. As $\Delta L \rightarrow 0$ (SPM) it tends to $i_d = 0$, recovering the textbook rule.

For an SPM held at $i_d = 0$, base speed is where the voltage limit is reached at full current: $\omega_{\text{base}} = V_{\max}/\sqrt{\psi_m^2 + (L I_{\max})^2}$. If the characteristic current ψ_m/L_d is below $I_{\max}$, the ellipse center lies inside the current circle and field weakening can in principle continue to very high speed.

6. Sensorless angle estimation

Medium and high speed, back-EMF. For an SPM in the stationary frame, $e_\alpha = -\omega_e \psi_m \sin \theta_e$ and $e_\beta = \omega_e \psi_m \cos \theta_e$, so $\hat{\theta}_e = \text{atan2}(-\hat{e}_\alpha, \hat{e}_\beta)$. The EMF is estimated by an observer from $v_{\alpha\beta} - R i_{\alpha\beta} - L di_{\alpha\beta}/dt$. IPMs use the "extended EMF" form. A PLL on the estimate usually replaces the raw atan2 for noise rejection.

Standstill and low speed, saliency. Inject a high-frequency voltage on the *estimated* d-axis. Because $L_d \neq L_q$, the high-frequency current that appears on the estimated q-axis is proportional to $(L_q - L_d) \sin 2\Delta\theta$, where $\Delta\theta$ is the angle error. Demodulate it and drive it to zero. (Magnet polarity is ambiguous by 180° and needs a separate check at start-up.)

7. Self-test, worked

Exercise 1: electrical vs. mechanical frequency

Four pole pairs at 3000 rpm: $f_m = 3000/60 = 50$ Hz, so $f_e = pf_m = 200$ Hz ($\omega_e \approx 1257$ rad/s). A 20 kHz current loop gets $20,000/200 = 100$ samples per electrical cycle, and the rotor advances 3.6° electrical per sample. With about 1.5 samples of control delay the voltage lands about 5.4° behind where it was aimed, so drives advance the inverse-Park angle by $1.5 \omega_e T_s$. At 15,000 rpm the same loop has only 20 samples per cycle, which is why high-speed drives push switching frequency up (one reason SiC and GaN matter).

Exercise 2: why $i_d = 0$ wastes current in an IPM

Torque is $\frac{3}{2}p i_q (\psi_m + (L_d - L_q)i_d)$. With $L_q > L_d$, a negative i_d *increases* the bracket, so the same i_q makes more torque. Spending a little current on i_d buys more torque per total amp than putting it all into i_q , until the MTPA balance point above. In an SPM $(L_d - L_q) \approx 0$, so i_d contributes nothing and should be zero (below base speed).